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COMPARISON BETWEEN DIFFERENT GROWTH FUNCTIONS OF THE JATROPHA CURCAS PLANT WITH RANDOM ATTACK PATTERN OF WHITEFLY

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ABSTRACT

We have proposed here two deterministic models of Jatropha Curcas plant and Whitefly that recreate the dynamics of cooperation between them where the conveyance of Whitefly on plant follows Poisson distribution.In the first model growth rate of the plant is thought to be in logistic form whereas in the second model it is taken as exponential form. The attack pattern and the growth of the whitefly are assumed as Holling type II function.The first model outcomes a globally stable state and in the second one we discover a globally attracting steady state for some parameter values,and a stable limit cycle for some other parameter esteems. It is likewise demonstrated that there exist Hopf bifurcation regarding some parameter values. The paper additionally examine the inquiry regarding persistence and permanence of the model. It is discovered that the particular growth rate of both the population and attack pattern of the whitefly administers the dynamics of both the models.

Keywords: *Jatropha curcas plant, Whitefly (Bemisia tabaci), Random attack , Global stability, Limit cycle and Hopf bifurcation investigation*.

I. INTRODUCTION

With the upgradation of human civilisation the interest for the elective vitality sources is also likewise expanding. Among the potential methods for creating vitality in a situation invitingway,the creation of biofuels is getting generally well known. Jatropha curcas is such a significant plant the seeds of which plant contains 37% oil that can be utilissed to obtain a superior nature of biodiesel [8]. The beginning of this plant is tropical zone at first from Mexico and central part of the USA and is currently developed overall [6]. The tree is of critical financial significance for its various mechanical and medicinal use.

Jatropha curcas is a semi-evergreen little bush with huge green to light green leaves. Normally it develops between $(3-5)$meter in tallness yet accomplishes a stature upto $(8-10)$meter under good condition. It is normally known as physic or purging nut. It is a multipurpose and drought resistant crop which is developed in marginal grounds with lesser input. The tree can be developed in dry and barren conditions and can be developed likewise in rough,sandy and salty soils. It has low plantation cost. It developes rapidly and lives delivering seeds for 50 years. Yet such a significant plant is influenced by the mosaic virus (begomovirus).

Mosaic virus is one kind of plant virus that causes the leaves of plants with a spotted and speckled look. They move oftentimes in nature. The indications are serious mosaic, mottling, blistering of leaves, yellowing of leaves, decreased leaf size, hindering of infected plants. It basically attacks its fruits extensively decreasing the creation and nature of seeds. The mosaic virus spreading chiefly relies upon the vector whitefly [2][3] . The number of inhabitants in whitefly is constrained by temperature and rainfall. Heavy rainfall makes an obstacle for the develoment of whiteflies [1]. In this ailment the mosaic virus passes from an infected whitefly to a susceptible plant and the other way around. The spread of the virus is profoundly depends on the plant thickness. A solitary whitefly is satisfactory to contaminate the host plants however transmission of the ailment spread when various infected whiteflies feed on the host plants through massive flux of saliva. Accordingly have host plant (Jatropha Curcas) faces leaf harm and sap seepage because of such feeding. Whiteflies reproduces very quickly [7] , if once they get traditional on any aspect of the plants they will willfully roam and try to attempt to assault some other immediate vegetation [5][10]. Ordinarily they need 3 hours feeding time to secure the infection and a latent phase of 8 hours. It requires 10 minutes time to taint the youthful leaves. Indications appear to be showed up after a latent period of 3-5 weeks. Also the infected whiteflies infuse the infection to the plant with the disease being almost certain if more insects assault the same plant. After obtaining of the mosaic virus adult whiteflies can infect the host plants within 48 hours. The motivation behind this study is to show the dynamics because of different growth function of the plant population and the effect of random attack pattern of whitefly on the plant [9] . A PC result shows the behavior of the solutions for various parameter esteems.

II. STATEMENT OF THE MODEL

In our model we have thought that v whiteflies are distributed over x plants so that some plants are without whitefly and others have 1,2,............,i whiteflies per plant. Consequently we have

$$\sum_{i=1}^{x} i = v$$

we here accepted that the whiteflies are conveyed over x plants as indicated by a probability distribution so that the extent of plants with i whiteflies is p(i). So the quantity of plants with i whiteflies is p(i)x. If the intrinsic plant loss-rate per whitefly is f then the loss-rate of plants with i whiteflies will be fip(i)x. Thus the total loss-rate of plants is

$$fx\sum_{i=0}^{\infty} ip(i)$$

.

Here $\sum ip(i)$ is the mean number of whitefly per plant and v/x is the expectation of i. so the loss rate due to whitefly consumption is fv. The loss of whiteflies occur in the following ways.

e-natural mortality of whitefly.

b=natural mortality of the host plant.

f=by their killing the host plant.

This self induced mortality occurs at a rate $fi^2p(i)x$. So for the whole plant population it is

$$fx\sum_{i=0}^{\infty} i^2p(i)$$

The term $\sum i^2p(i)$ is the expectation of i^2. We have chosen here the poisson distribution which ecologically reflects random attack pattern. Here whitefly-inflicted losses through the plant death are $fxE(i^2)$.

For poisson distribution we have $E(i^2) = \frac{v}{x} + (\frac{v}{x})^2$ [9]. We have picked two different types of growth function of the plant(Jatropha curcas) population. In the first model we have picked the growth of the plant population in logistic form and in the second model exponential growth is accepted. In both the model the growth of whitefly and the attack pattern of the whitefly is accepted as holling type II function. Here r is growth rate of the whitefly,k is the carrying capacity.

Based on the above suppositions the first model takes the structure,

III. MODEL 1

$$\frac{dx}{dt} = rx(1-\frac{x}{k}) - \frac{axv}{k+x}$$
$$\frac{dv}{dt} = v[\frac{cx}{k+x} - (e+b+f) - \frac{fv}{x}] \qquad (1)$$

with the initial conditions,

$x(0) = x_0 > 0, v(0) = v_0 > 0$

Here x_0 is the initial plant population density and v_0 is the initial whitefly density.

For mathematical convenience we consider the following transformation,

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$x = kX$, $v = \frac{kcv}{f}$, $t = \frac{\tau}{c}$.

The transformed equation is,

$$\frac{dX}{d\tau} = \alpha X(1-X) - \frac{\beta XV}{1+X}$$
$$\frac{dV}{d\tau} = V[\frac{X}{1+X} - \gamma - \frac{V}{X}]$$

where $\alpha = \frac{r}{c}$, $\beta = \frac{a}{f}$, $\gamma = \frac{b+e+f}{c}$. (2)

3.1 Solution properties

3.1.1 lemma1

The solution of (2) are positive.

Proof:

since $x(0) = x_0 > 0$ and $v(0) = v_0 > 0$, we have $X(0) = X_0 > 0$ and $V(0) = V_0 > 0$. Suppose $X(\tau)$ is not positive for all $\tau \geq 0$. Since $X_0 > 0$ then there exist τ_0 with $X(\tau_0) = 0$ and $X(\tau) > 0$ for $0 \leq \tau \leq \tau_0$. For $0 \leq \tau \leq \tau_0$

$$\frac{\dot{X}(\tau)}{X(\tau)} = \alpha(1-X) - \frac{\beta V}{1+X} > -\alpha X - \frac{\beta V}{1+X}$$

$$X(\tau_0) > X_0 exp[-\int_0^{\tau_0} V(\eta)/X(\eta)d\eta] > 0$$

This is a contradiction and hence $X(\tau)$ is positive for all $\tau \geq 0$. Similarly it can be shown that $V(\tau)$ is also positive for all $\tau \geq 0$.

3.2 Equilibria

The equilibrium points are obtained by setting $\frac{dX}{d\tau} = 0$ and $\frac{dV}{d\tau} = 0$ and solving the equations

$\alpha(1-X) - \frac{\beta V}{1+X} = 0$ and

$\frac{X}{1+X} - \gamma - \frac{V}{X} = 0.$

We have seen that there are two equilibrium points i.e. $E_1(X,0) = (1,0)$ which is the whitefly free equilibrium and $E_2(X^*,V^*)$ is the interior equilibrium. From the first equation we obtain V^* as a function of X^*, as follows.

$$V^* = \frac{\alpha(1-X)(1+X)}{\beta}$$

Clearly V^* is feasible as $X^* \leq 1$. Substituting this in the second equation we have a cubic equation as,

$$\alpha X^3 + (\beta - \beta\gamma + \alpha)X^2 - (\alpha + \beta\gamma)X - \alpha = 0$$

Since there is atleast one change of sign therefore by Descartes' rule of sign there exist atleast one positive X^*. Therefore (X^*,V^*) exists.

3.3 Stability

The equilibrium E_1 is stable if $\gamma > 0.5$ or saddle if $\gamma < 0.5$ as its eigen values are -α and $0.5 - \gamma$.

The characteristic equation for $E_2(X^*,V^*)$ is a quadratic equation which is as follows,

$$\lambda^2 + \lambda(-\alpha + 2\alpha X^* + \frac{\beta V^*}{(1+X^*)^2} + \frac{V^*}{X^*}) - \frac{\alpha V^*}{X^*} + 2\alpha V^* + \frac{\beta V^{*2}}{X^*(1+X^*)^2} + \frac{\alpha X^*(1-X^*)}{(1+X^*)^2} + \frac{\beta V^{*2}}{X^*(1+X^*)} = 0$$

Which can be written as

$$\lambda^2 + A\lambda + B = 0$$

Where,

$$A = \frac{2\alpha X^{*2}}{(1+X^*)} + \frac{V^*}{X^*} > 0$$

and

$$B = \frac{\alpha^2(1-X^{*2})}{\beta} + \frac{\alpha^2(1-X^*)^2}{\beta X^*} + \frac{\alpha X^*(1-X^*)}{(1+X^*)^2} > 0$$

Since $A > 0$,$B > 0$, $E_2(X^*, V^*)$ is locally asymptotically stable.

3.4 Global stability

Let us consider $H(X,V) = \frac{1}{XV}$

then $H > 0$ as $X > 0$ and $V > 0$

Let $h_1(X,V) = \alpha X(1-X) - \frac{\beta XV}{1+X}$

and $h_2(X,V) = V[\frac{X}{(1+X)} - \gamma - \frac{V}{X}]$

therefore $\nabla(X,V) = \frac{\partial(h_1 H)}{\partial X} + \frac{\partial(h_2 H)}{\partial V}$

$$= \frac{\partial \frac{\alpha(1-X)}{V} - \frac{\beta}{(1+X)}}{\partial X} + \frac{\partial \frac{1}{(1+X)} - \frac{\gamma}{X} - \frac{V}{X^2}}{\partial V}$$

$$= \frac{-2\beta X}{(1+X)^2(1-X)} - \frac{1}{X^2} < 0$$

Subsequently by Bendixson-Dulac criteria E_2 is globally asymptotically stable in the positive XV-plane [4] .

IV. PERSISTENCE AND PERMANENCE OF THE SYSTEM

From the biological point of view persistence impliess that all the populations are available and none of them will get terminated. Persistence and permanence is consistent to settle the inquiries of endurance and elimination of n-species whose growth equations are administered by the differential equations

$$\dot{x}_i = x_i f_i(x_1, x_2, \dots\dots\dots, x_n) \qquad (3)$$

The thought of persistence (weak and strong) came to the light by Freedman and Waltman. The system (2) is supposed to be weakly persistent if $limsup x_i(t) > 0$ for all orbits in $int\mathbb{R}^n_+$and strongly persistent if $liminf x_i(t) > 0$.

Again system (2) is said to be permanent if there exists a compact set $B \subset int\mathbb{R}^n_+$ such that all orbits in $int\mathbb{R}^n_+$ end up in B. The system is uniformly persistence if there exist $\delta > 0$ such that for each compact set x_i , $liminf x_i(t) \geq \delta > 0$ for all $(x_1(t), x_2(t), \dots\dots\dots, x_n(t)) = X(t) \in int\mathbb{R}^n_+$. We now discuss the concept of saturated equilibria. An equilibrium fixed point x^* is said to be saturated equilibrium if $x_i^* = 0$ then $f_i(x_1^*, x_2^*, \dots\dots\dots, x_n^*) \leq 0$. With the concept of saturated equilibria and by the method of average Lyapunov function we have the following theorem for permanent coexistence of both the species of the system [4].

4.1 Theorem

The system is permanent iff $\gamma < \frac{1}{2}$.

Proof:

The index theorem states that the system with dissipativeness suspicion has atleast one saturated equilibrium. If all these saturated equilibria are regular, then the sum of their indices is +1. From the lemma 1 the system is dissipative and thus there exists atleast one saturated equilibrium and the sum of their indices is +1 if they are regular. The permanence of the system implies that none of the boundary fixed points are saturated. Hence the interior fixed point exists and must be saturated. Hence all the eigen values are negative or have negative real part,which is conceivable if $\gamma < \frac{1}{2}$.

We now build the average Lyapunov function to prove the sufficient condition. In our model, we consider the average Lyapunov function as $\sigma(X) = X^{r_1}.V^{r_2}$ where $r_i > 0$ i=1,2.

Let, $\psi(X) = \frac{\dot{\sigma}(X)}{\sigma(X)}$

$$= r_1\frac{\dot{X}}{X} + r_2\frac{\dot{V}}{V}$$

$$= r_1[\alpha(1-X) - \frac{\beta V}{(1+X)}] + r_2[\frac{X}{(1+X)} - \gamma - \frac{V}{X}]$$

If $\psi(X) > 0$ for the ω -limit sets of trajectories initiated in $\mathbb{R}_+^3$, then the trajectories more away from the boundary and the system (1) is permanent. It is evident that there is no periodic trajectory. Hence if there exist $r_1 > 0$ such that $\Psi(E_1) > 0$, then (1) is permanent.

Therefore for $E_1(1,0)$, $\psi(X) = r_2(\frac{1}{2} - \gamma) > 0$

The inequality is evidently satisfied for atleast one positive $r = (r_1, r_2)$ if $\gamma < \frac{1}{2}$. Henceforth the system is uniformly persistent(or permanent) if $\gamma < \frac{1}{2}$. This completes the proof of the theorem.

V. MODEL 2

In this model keeping all the things same as model 1 we have taken the exponential growth of the plant (Jatropha curcas). Now our model 2 is as follows:

$$\frac{dx}{dt} = rx - \frac{axv}{k+x}$$
$$\frac{dv}{dt} = v[\frac{cx}{k+x} - (e+b+f) - \frac{fv}{x}] \qquad (4)$$

For convenience we have chosen the dimensionless form by taking,

$x = kX, v = \frac{kc}{a}V, t = \frac{\tau}{c}$

The dimension less form becomes:

$$\frac{dX}{d\tau} = \alpha X - \frac{XV}{1+X}$$
$$\frac{dV}{d\tau} = V[\frac{X}{1+X} - \beta - \frac{\gamma V}{X}] \qquad (5)$$

where $\alpha = \frac{r}{c}$, $\beta = \frac{e+f}{c}$, $\gamma = \frac{f}{a}$.

5.1 Equilibria

The equilibrium points can be obtained by setting $\frac{dX}{d\tau} = 0$, and $\frac{dV}{d\tau} = 0$

we here observed that there is just a single equilibrium point $E(X^*, V^*)$ i.e. the interior equilibrium point.

From the first equation of (5) we get V^* in terms of X^* which is as follows:

$$V^* = \alpha(1 + X^*)$$

substituting this in the second equation X^* is obtained as:

$$X^* = \frac{(\beta + 2\alpha\gamma) \pm \sqrt{(\beta + 2\alpha\gamma)^2 + 4(1-\beta-\alpha\gamma)\alpha\gamma}}{2(1-\beta-\alpha\gamma)}$$

So X^* exists if $\beta + \alpha\gamma < 1$

5.2 Stability

The local behavior of the equilibrium point of the system is determined by the real parts of the eigenvalues of the Jacobian matrix at that point.

The characteristic equation is given by:

$$\lambda^2 + \lambda(-\frac{\alpha X^*}{1+X} + \frac{\gamma V}{X}) - \frac{\alpha\gamma V}{1+X} + \frac{\alpha X}{(1+X)^2} + \frac{\gamma V^2}{X(1+X)} = 0$$

Which can be written as:

$$\lambda^2 + A\lambda + B = 0$$

Where $A = \frac{X-\beta-\beta X-\alpha X}{1+X}$
and $B = -\frac{\alpha\gamma V}{1+X} + \frac{\alpha X}{(1+X)^2} + \frac{\gamma V^2}{X(1+X)} > 0$
This leads to the following results for $A > 0$ or $A = 0$ or $A < 0$:

5.2.1 Theorem:
If $\beta + \alpha\gamma < 1$ and $X^* - \beta - \beta X^* - \alpha X^* > 0$ then $E(X^*, V^*)$ is globally asymptotically stable.

Proof:
If possible let Γ be any periodic orbit around $E(X^*, V^*)$ in the positive XV- plane. Then,

$$\Delta = \int_\Gamma div(\dot{X}, \dot{V})\, d\tau$$
$$= \int_\Gamma \left(\alpha - \frac{V}{(1+X)^2} + \frac{X}{(1+X)} - \beta - \frac{2\gamma V}{X}\right) d\tau$$
$$= \int_\Gamma \left(\frac{\alpha X}{(1+X)} - \frac{\gamma V}{X}\right) d\tau$$

Under the given assumption $E(X^*, V^*)$ is locally stable. In this way $\Delta < 0$. The Poincare criteria now suggests that the proposed periodic orbit Γ is stable , which leads to a logical inconsistency. Accordingly, there is no periodic orbit around $E(X^*, V^*)$ in the positive XV plane and thus $E(X^*, V^*)$ is a global attractor. This completes the proof of the theorem.

5.2.2 Theorem:
If $\beta + \alpha\gamma < 1$ and $X^* - \beta - \beta X^* - \alpha X^* = 0$ then the system bifurcates into small amplitude periodic solutions near $E(X^*, V^*)$.
Proof:
To demonstrate this theorem we can show that the conditions for a hopf bifurcations are fulfilled. If $X^* - \beta - \beta X^* - \alpha X^* = 0$ and the two roots of the characteristic equation $\lambda^2 + A\lambda + B = 0$ are purely imaginary namely $\pm i\eta$.

where $\eta^2 = \frac{-\alpha V\gamma}{(1+X)} + \frac{\alpha X}{(1+X)^2} + \frac{\gamma V^2}{X(1+X)}$.

The necessary and sufficient condition for hopf bifurcation to occur is that there exist a $\gamma = \gamma^*$ such that

i)$X^* - \beta - \beta X^* - \alpha X^* = 0$ and
ii)$\frac{d(Real\lambda)}{d\gamma}|_{\gamma=\gamma^*} \neq 0$

Henceforth all the conditions for a Hopf bifurcation are fulfilled. This finisheses the confirmation of the theorem.

5.2.3 Theorem:
If $\beta + \alpha\gamma < 1$ and $X^* - \beta - \beta X^* - \alpha X^* < 0$ then there exists a stable limit cycle around $E(X^*, V^*)$ in the positive XV plane.
Proof:
Now if we decrease the value of $X^* - \beta - \beta X^* - \alpha X^*$ such that
$X^* - \beta - \beta X^* - \alpha X^* < 0$ then $E(X^*, V^*)$ is locally unstable. Again

$$\Delta = \int_\Gamma div(\dot{X}, \dot{V})\, d\tau$$
$$= \int_\Gamma \left(\alpha - \frac{V}{(1+X)^2} + \frac{X}{(1+X)} - \beta - \frac{2\gamma V}{X}\right) d\tau$$
$$= \int_\Gamma \left(\frac{\alpha X}{(1+X)} - \frac{\gamma V}{X}\right) d\tau$$

So, we can reason that $\Delta > 0$ if $X^* - \beta - \beta X^* - \alpha X^* < 0$. Thus by Poincare criteria any periodic orbit is stable. Thus there exists atleast one stable limit cycle around $E(X^*, V^*)$ in the positive XV plane.

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VI. NUMERICAL SIMULATION AND DISCUSSIONS

To check the theoretical outcomes numerical simulations have been completed utilizing MATLAB-2016a. Here we have utilized MATLAB routine ODE23. In this numerical simulation we have used distinctive permissible estimations of the system parameters to ensure our theoretical results. For the model 1, we have chosen a set of parameter values such as $\alpha = 0.75$, $\beta = 2$, $\gamma = 0.2$ that shows the local as well as global stability which also ensures the theoretical results.The equilibrium point corresponding to this set of parameter values of model 1 is (0.7429,0.168037346).

For the model 2, Keeping in mind the feasibility criteria we have chosen the values of γ by using the following conditions,

i)$X^* - \beta - \beta X^* - \alpha X^* > 0$
ii)$X^* - \beta - \beta X^* - \alpha X^* = 0$
iii)$X^* - \beta - \beta X^* - \alpha X^* < 0$

For the set of parameter values $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.35$ satisfying the condition $X^* - \beta - \beta X^* - \alpha X^* > 0$ the equilibrium point becomes (1.209914074,1.546939852). The corresponding figure shows locally steady state which leads to global stability around the equilibrium point. The corresponding phase portrait for the same set of parameter values also ensures the same by figure 3 and 4.

For the set of parameter values $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.111111111$ satisfying the condition $X^* - \beta - \beta X^* - \alpha X^* = 0$ the equilibrium point becomes (1.124650736,1.487255515) . The corresponding figure shows small amplitude oscillation which leads to Hopf bifurcation around the equilibrium point. The corresponding phase portrait of this Hopf bifurcation of the system (4) has been represented in the figure 5 and 6 for the same set of parameter values.

For the set of parameter values $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.07$ satisfying the condition $X^* - \beta - \beta X^* - \alpha X^* < 0$ the equilibrium point is (0.383003659,0.968102561) which locally shows the unstable behavior. It is observed that there is a large amplitude oscillation with increasing time for both the plant and whitefly which leads to limit-cycle. The corresponding phase portrait of this stable limit- cycle of the system (5) has been shown in the figure 7 and 8.
In the realistic situation we also observe the same phenomena.

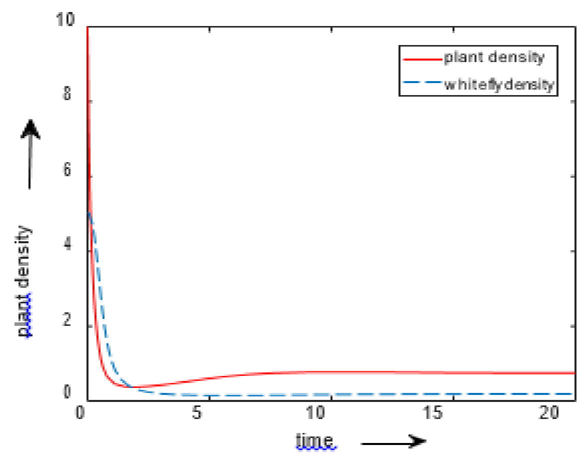

Figure 1: Variation of plant-herbivore densities with time in model 1 for $\alpha = 0.75$, $\beta = 2$, $\gamma = 0.2$. Here we observe local stability for the population with increasing time.

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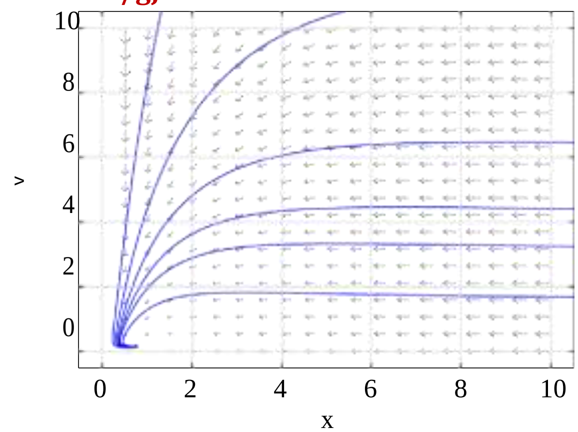

Figure 2: Variation of plant-herbivore densities in model 1 $\alpha = 0.75, \beta = 2, \gamma = 0.2$ .This shows the phase portrait in the XV plane which is globally asymptotically stable state of model 1.

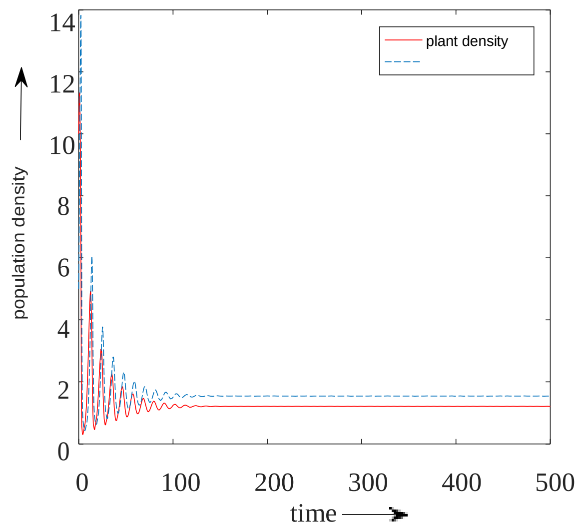

Figure 3: Locally asymptotically stable state for both the population for the set of parameter values $\alpha = 0.7, \beta = 0.1, \gamma = 0.35$ for model 2.

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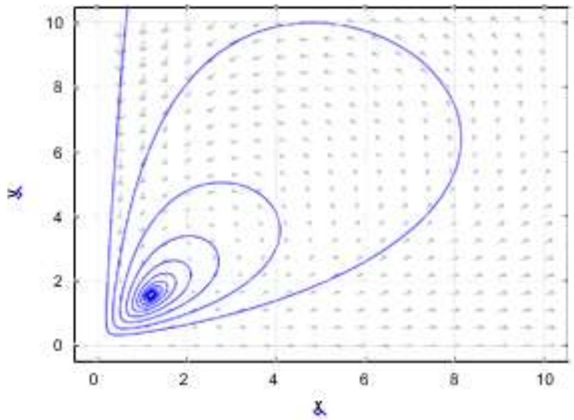

Figure 4: Phase portrait for the plant-herbivore system with the same parameter values $\alpha = 0.7$ $\beta = 0.1$, $\gamma = 0.35$ ***which shows global asymptotic stability of model 2.***

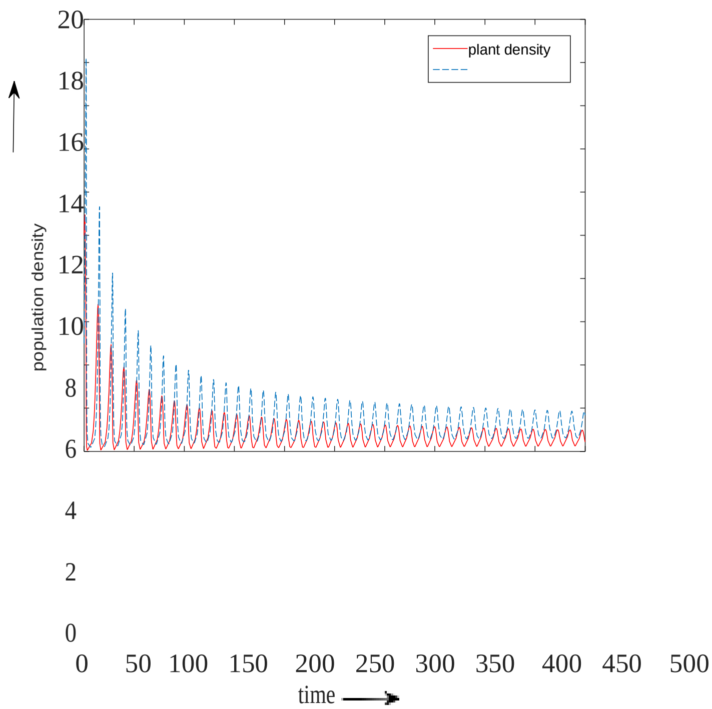

Figure 5: Small amplitude oscillations of both the population for the set of parameter values $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.111111111$ ***of model 2.***

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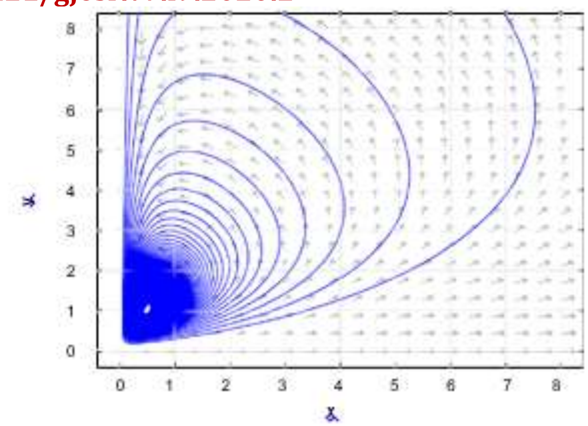

Figure 6: Hopf bifurcation for the parameter values $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.111111111$ of model 2

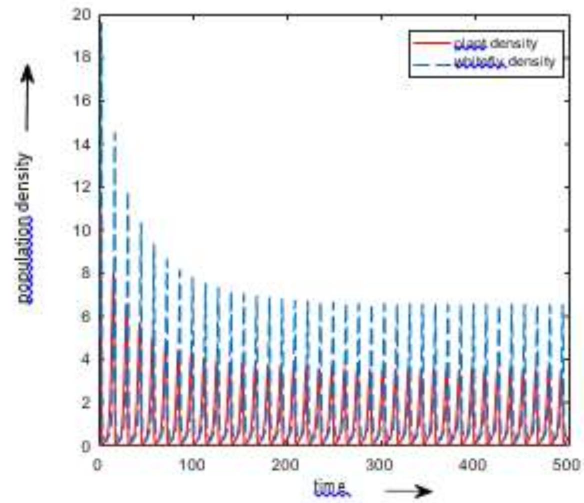

Figure 7: Large amplitude oscillations of both the population which indicates unstable condition as time increase for the set of parameter values $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.07$ of model 2.

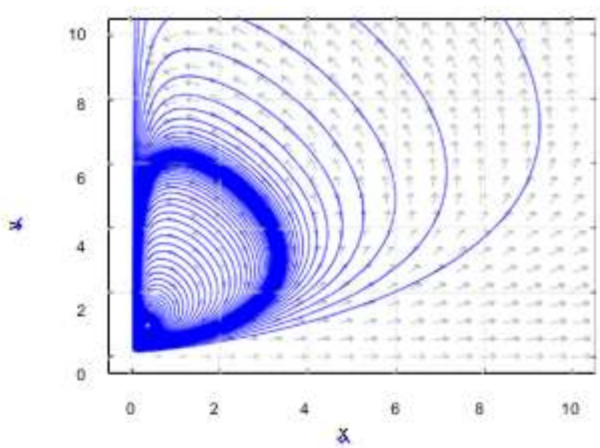

Figure 8: Stable limit-cycle for the parameter values $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.07$.

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VII. CONCLUSION

This investigation is based on the association between Jatropha curcas plant and the vector whitefly. Here a comparison of two distinct growth function of the Jatropha curcas plant is represented with random attack pattern of the whitefly using poisson distribution. From our study it is explicit that if the plant grows logistically then the effect of whitefly cannot destabilize the system but if the plant growth is exponential then it shows three different types of behavior depending upon the different parameter values. It shows global stability for some parameter values, hopf bifurcation for some other parameter values and stable limit cycle for some another set of parameter values. Our mathematical outcomes additionallyl underpins the similar conduct.

VIII. ACKNOWLEDGMENTS

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